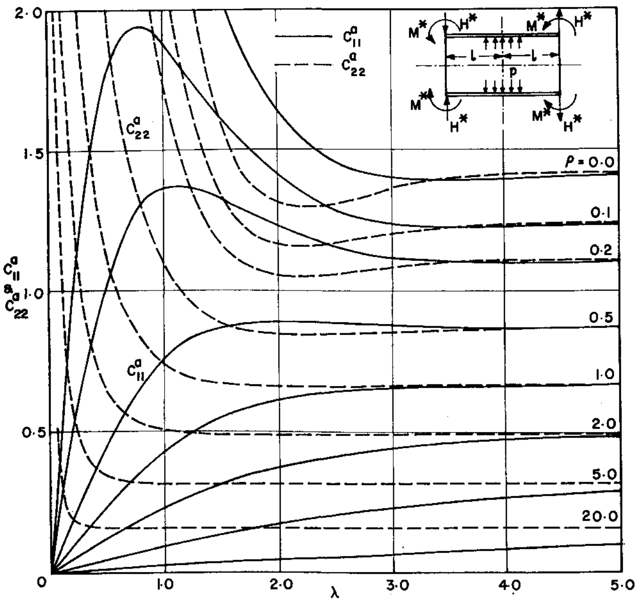

Fig. 6 Influence coefficients C_{11}^a and C_{22}^a for antisymmetric edge loads as functions of λ.

for $\rho = 1$:

$$C_{11}^a = \frac{2\alpha[\cosh 2\alpha\lambda - 1]}{[(1 + 2\rho)\sinh 2\alpha\lambda - 2\alpha\lambda(1 - 2\rho)]} \quad \text{(11a)}$$

$$C_{12}^a = C_{21}^a = \frac{[\sinh 2\alpha\lambda + 2\alpha\lambda]}{[(1 + 2\rho)\sinh 2\alpha\lambda - 2\alpha\lambda(1 - 2\rho)]} \quad \text{(11b)}$$

$$C_{22}^a = \frac{2\alpha[\cosh 2\alpha\lambda + 1]}{[(1 + 2\rho)\sinh 2\alpha\lambda - 2\alpha\lambda(1 - 2\rho)]} \quad \text{(11c)}$$

for $\rho > 1$:

$$C_{11}^a = \frac{2\alpha\delta[\cosh 2\alpha\lambda - \cosh 2\delta\lambda]}{[\delta(1 + 2\rho)\sinh 2\alpha\lambda - \alpha(1 - 2\rho)\sinh 2\delta\lambda]} \quad \text{(12a)}$$

$$C_{12}^a = C_{21}^a = \frac{[\alpha \sinh 2\delta\lambda + \delta \sinh 2\alpha\lambda]}{[\delta(1 + 2\rho)\sinh 2\alpha\lambda - \alpha(1 - 2\rho)\sinh 2\delta\lambda]} \quad \text{(12b)}$$

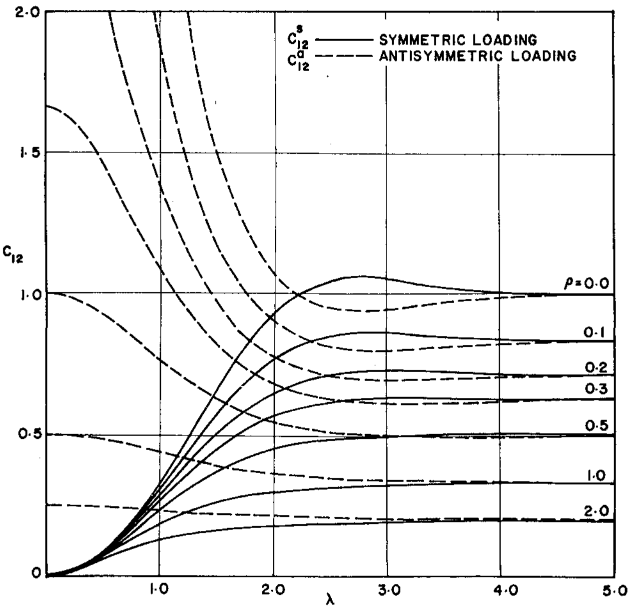

Fig. 7 Influence coefficient C_{12} for symmetric and antisymmetric edge loads as function of λ.

$$C_{22}^a = \frac{2\alpha\delta[\cosh 2\alpha\lambda + \cosh 2\delta\lambda]}{[\delta(1 + 2\rho)\sinh 2\alpha\lambda - \alpha(1 - 2\rho)\sinh 2\delta\lambda]} \quad \text{(12c)}$$

where α, β and δ are given by Eq. (8).

Numerical Results and Conclusions

Numerical calculations of the influence coefficients given by Eqs. (5–7) and (10–12) were done on a CDC 3600 computer for various values of the pressurization parameter ρ and length parameter λ. Figures 5, 6, and 7 show the graphs of the influence coefficients as a function of λ for various specified values of the parameter ρ, up to a value of $\lambda = 5.0$. Beyond this value of λ, the length effect becomes negligible and the values of the influence coefficients approach that for a semi-infinite cylindrical shell under axisymmetric loads at its edge.[2] For $\rho = 0$ the expressions for the influence coefficients given here agree with those given in Ref. 3 if due allowance is made for the sign conventions and notations.

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Free Vibrations of an Isotropic Nonhomogeneous Circular Plate

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Introduction

Papers pertaining to vibrations of the nonhomogeneous plate[1,2] or in different context, vibrations of plates of variable thickness[3,4] are not found in abundance in literature.

Z. Mazuriewiez[1,2] in his two papers discussed the vibration of the nonhomogeneous rectangular plate. There the general problem was formulated in integral form and then reduced to an infinite system of linear homogeneous equations. The eigenvalues appear as roots of an infinite determinant and therefore the method does not afford exact values. This Note has discussed the transverse vibration of nonhomogeneous free circular plate. Nonhomogeneity of the plate is characterized by taking

$$E = E_0(1 - \rho^2)^\alpha, \quad \sigma = \sigma_0(1 - \rho^2)^{\alpha-2}, \quad (0 \leq \rho \leq 1)$$

where $\rho = r/a$, a the radius of the disc, E and σ are Young's Modulus and density, respectively, of the plate and α (integer > 3) is the index of nonhomogeneity. Explicit closed-form expressions are found for nodal frequencies, and effect of nonhomogeneity is shown in tabular form. The $\alpha = 3$ case coincides with Harris.[4]

Basic Equation

The equation of motion in polar coordinates for small deflection of plate[4] is

$$\sigma h \partial^2 W/\partial t^2 = (1 - \nu)\diamondsuit^4(D,W) - \nabla^2(D\nabla^2 W) \quad (1)$$

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Table 1 Values of frequency Ω_{mn}; $\alpha = 4, \nu = 0.3$

m/n	0	1	2	3	4	5	6
0	0	12.90	37.09	71.08	109.17	157.17	213.18
1	0	23.00	51.10	87.71	131.16	183.17	243.18
2	8.20	32.93	65.06	105.11	139.35	209.16	273.17
3	14.20	41.80	79.01	123.08	175.11	235.13	303.16
4	20.02	52.76	92.89	141.03	197.08	261.39	333.28
5	25.92	62.66	106.88	158.98	219.04	287.08	363.16
6	31.74	72.55	120.81	176.93	241.04	313.05	393.08

where

$$\diamondsuit^4(D,W) = \frac{\partial^2 D}{\partial r^2}\left(\frac{1}{r}\frac{\partial W}{\partial r} + \frac{1}{r^2}\frac{\partial^2 W}{\partial \theta^2}\right) + \frac{\partial^2 W}{\partial r^2}\left(\frac{1}{r}\frac{\partial D}{\partial r}\right) \quad (2)$$

W, D, σ, h, ν are respectively deflection, rigidity, density, thickness and Poisson's ratio of the plate.

We consider a circular plate of radius a. The variable Young's modulus and density of the plate are given by

$$E = E_0(1-\rho^2)^\alpha \qquad (\alpha \text{ integer} > 3)$$
$$\sigma = \sigma_0(1-\rho^2)^{\alpha-2}$$

where E_0,σ_0 are constants. Therefore the rigidity of nonhomogeneous plate with constant thickness h and constant Poisson's ratio ν is given by

$$D = E_0 h(1-\rho^2)^\alpha/12(1-\nu^2) \quad (3)$$

At the plate boundary ($\rho = 1$), it follows that

$$D = \partial D/\partial r = 0 \quad (4)$$

and the free edge boundary conditions are automatically satisfied.

Substituting Eq. (3) in Eq. (1) and assuming the solutions of the resulting equations in the form

$$W = e^{i\omega t} f_m(\rho)\cos m\theta \quad (5)$$

we get

$$\left[(1-\rho^2)^2\frac{\partial}{\partial\rho^2} + (1-\rho^2)\{1-(4\alpha+1)\rho^2\}\frac{1}{\rho}\frac{\partial}{\partial\rho} - 4\alpha(1-\alpha\rho^2) - (m^2/\rho^2)(1-\rho^2)^2\right] \times \left[\frac{1}{\rho}\frac{df_m}{\partial\rho} + \frac{\partial^2 f_m}{\partial\rho^2} - (m^2/\rho^2)f_m\right] + 2\alpha(1-\nu)\left[(1-\rho^2)\frac{\partial^2 f_m}{\partial\rho^2} + \{1-(2\alpha-1)\rho^2\}\left(\frac{1}{\rho}\frac{\partial f_m}{\partial\rho} - \frac{m^2}{\rho^2}f_m\right)\right] = \Omega^2 f_m(\rho) \quad (6)$$

where

$$\Omega^2 = \sigma_0 a^4 h\omega^2/D_0 \quad (7)$$

To find the solution of the differential equation let us take $f_m(\rho)$ in the series form as

$$f_m(\rho) = \sum_{s=0}^{\infty} a_s \rho^{m+2s} \quad (8)$$

Table 2 Values of frequency Ω_{mn}; $\alpha = 5, \nu = 0.3$

m/n	0	1	2	3	4	5	6
0	0	18.54	46.21	81.97	125.79	177.65	237.40
1	0	29.59	61.70	101.64	179.58	205.47	269.40
2	10.58	40.98	77.30	121.35	173.34	233.30	301.26
3	18.33	52.49	92.95	139.95	162.22	261.14	333.13
4	25.92	64.34	108.62	160.84	220.94	288.97	365.00
5	33.18	75.65	124.36	180.61	244.75	316.67	396.89
6	40.98	87.27	140.03	200.17	268.56	344.67	415.08

Table 3 Values of frequency Ω_{mn}; $\alpha = 10, \nu = 0.3$

m/n	0	1	2	3	4	5	6
0	0	32.25	80.69	135.63	200.87	311.25	349.00
1	0	53.87	105.72	166.63	234.74	331.81	394.84
2	22.47	74.77	131.72	196.14	268.37	348.51	458.94
3	38.89	95.41	156.79	225.46	301.84	386.10	478.27
4	55.00	115.93	181.63	254.65	335.26	423.58	519.84
5	71.00	146.32	206.57	283.76	368.49	461.00	562.53
6	86.95	160.33	231.36	312.91	401.12	498.35	601.99

Substitution of Eq. (8) into Eq. (6) yields[4]

$$\rho^m(1,\rho^2,\rho^4,\ldots)\times\{A_m + 2\alpha(1-\nu)B_m - \Omega^2 I\}\begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \end{pmatrix} = 0 \quad (9)$$

where A_m and B_m are upper triangular matrices defined as

$$A_m = \begin{bmatrix} \gamma_{m0} & \beta_{m1} & \alpha_{m2}', & 0\ldots & & & \\ 0 & \gamma_{m1} & \beta_{m2} & \alpha_{m3}' & 0\ldots & & \\ . & 0 & \gamma_{m2} & \beta_{m3} & \alpha_{m4}' & 0\ldots & \\ . & & & & & & \\ . & & & & & & \end{bmatrix}$$

$$B_m = \begin{bmatrix} \epsilon_{m0} & \delta_{m1} & 0 & \ldots & & \\ 0 & \epsilon_{m1} & \delta_{m2} & 0 & \ldots & \\ 0 & 0 & \epsilon_{m2} & \delta_{m3} & 0 & \ldots \\ . & & & & & \\ . & & & & & \\ . & & & & & \end{bmatrix}$$

and I is the unit matrix.

The elements of the matrices are given as follows:

$$\alpha_{ms}' = 16s(m+s)(s-1)(m+s-1)$$
$$\beta_{ms} = 16s(m+s)[2ms + 2s^2 + (\alpha-2)(m+2s-1)]$$
$$\gamma_{ms} = 16s(m+s)(s+\alpha-1)(m+s+\alpha-1)$$
$$\delta_{ms} = 4s(m+s)$$
$$\epsilon_{ms} = 4[\tfrac{1}{2}(\alpha-1) - ms - s^2 - \tfrac{1}{2}(\alpha-1)m + (\alpha-1)s]$$

Since the matrix is upper triangular, its eigenvalues are its diagonal elements. The nodal frequency is given by

$$\Omega_{mn}^2 = \alpha_{mn}' + 2\alpha(1-\nu)\delta_{mn}$$

This expression is in closed form; it depends upon Poisson's ratio and degree of nonhomogeneity.

Numerical Results

Below Tables 1–3 are given, taking values of $\nu = 0.3$ and $\alpha = 4,5,10$. It is evident that the frequency modes are definitely affected by the degree of nonhomogeneity.

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